TOWARD SIMPLIFICATION OF DYNAMIC SUBGRID-SCALE MODELS

C. David Pruett*
Department of Mathematics
James Madison University
Harrisonburg, VA, USA 22807

Abstract

We examine the relationship between the filter and the subgrid-scale (SGS) model for large-eddy simulations, in general, and for those with dynamic SGS models, in particular. From a review of the literature, it would appear that many practitioners of LES consider the link between the filter and the model more or less as a formality of little practical effect. In contrast, we will show that the filter and the model are intimately linked, that the Smagorinsky SGS model is appropriate only for filters of first- or second-order, and that the Smagorinsky model is inconsistent with spectral filters. Moreover, the Germano identity is shown to be both problematic and unnecessary for the development of dynamic SGS models. Its use obscures the following fundamental realization: *For a suitably chosen filter, the computible resolved turbulent stresses, properly scaled, closely approximate the SGS stresses.*

1 Introduction

By definition, direct numerical simulation (DNS) is the numerical solution of the Navier-Stokes equations without recourse to empirical models. In concept, the fluid motions are resolved down to the Kolmogorov length scale, at which eddies succumb to viscous dissipation. In general, the computational workload for fully-resolved DNS scales as Re^3, where Re is the Reynolds number. Consequently, for the complex, high-Reynolds-number flows of engineering interest, the computational requirements DNS are staggering and prohibitive.

In contrast, in large-eddy simulation (LES), the larger scales of motion are resolved in space and time on a moderately coarse grid; however, the effect of the subgrid-scale (SGS) motions on the evolution of the larger scales is modeled. In practice, the decomposition into resolved and unresolved scales is accomplished by a spatial (temporal) filtering operation with an associated cutoff length (time) scale Δ.

First introduced in the 1960's, LES has experienced a resurgence of interest since 1991, when dynamic SGS modeling was proposed by Germano and coworkers[6]. The advantages and difficulties associated with dynamic SGS models are now well established, and space does not permit elaboration. However, it is fair to say that the promise of dynamic modeling has not been fully realized largely because many of the proposed fixes to the shortcomings of dynamic models involve considerable additional complexity and computational overhead.

Here, our purpose is to examine the connection between the choice of the filter and the subgrid-scale (SGS) model, with an eye toward the simplification of dynamic SGS models.

From a review of the literature, it would appear that many practitioners of LES consider the link between the filter and the model more or less as a formality of little practical effect. Surprisingly little is written on this topic, Piomelli et al.[9] and Aldama[1] excepted. Specifically, regarding the conventional practice of LES, Piomelli et al.[9] observed: "In the past, however, the choices of model and filter have been regarded as completely independent." Recognizing that the behavior of the SGS model strongly depends on the choice of

*Research conducted under NASA Grant NAG-1-1802, monitored by Dr. Kristine R. Meadows, NASA Langley Research Center, Hampton, VA 23681-0001

the filter, they attempted to address the issue of filter-model consistency on the basis of physical arguments and *a priori* tests, which involve comparisons of the exact and modeled SGS stresses computed by fully resolved DNS. Here, we approach filter-model consistency from a mathematical point of view. From the present analysis come a number of revelations, some of which run counter to conventional wisdom.

In the next section, we discuss aspects of linear filters in general, focusing on their order properties. In the third section, starting from the Germano identity and in the context of a dynamic SGS model, we derive a simple but accurate approximation for the residual stresses. In the fourth section, we derive a similar result without first appealing to the Germano identity. In the fifth section, we briefly discuss the implications of these results. Conclusions are summarized in the final section.

2 Linear Filter Operators

Like differential operators, filter operators may be either continuous or discrete. Here, for brevity, we consider only continuous filters (which are also referred to as "analog"); however, the conclusions drawn for continous filters generalize immediately to linear discrete filters. Moreover, also for brevity, we consider only time-domain filters. However, the implications should apply to spatial filters as well.

Let $f(t,\vec{x})$ be a continous function of time and space, and let Δ, the "window" width, denote a characteristic time scale associated with the temporal linear filter $F[f(t),\Delta]$. As a specific example, consider the continuous, causal filter given by the integral equation

$$F\left[f(t,\vec{x}),\Delta\right] = \frac{1}{\Delta}\int_{t-\Delta}^{t} f(\tau,\vec{x})d\tau \tag{1}$$

From first principles of the Calculus, it is readily shown that $\lim_{\Delta\to 0} F[f(t),\Delta] = f(t)$. On the other hand, for a finite window Δ, the time-domain filter above tends to remove oscillations of high frequency relative to Δ while preserving low-frequency oscillations, which defines a "low-pass" filter. For applications to LES, we consider only low-pass filters.

The effect of a filter is most apparent in Fourier space. To each filter is associated a transfer function $H(\Omega)$ that quantifies the amplitude and phase effects of the filter on oscillations of dimensionless frequency $\Omega = \omega\Delta$. For example, the transfer function associated with Eq. 1, shown in Fig. 1, is readily obtained by directly integrating $F(e^{i\omega t},\Delta)$ for arbitrary ω. Figure 1 reveals some undesirable traits of the filter: the amplitude decay is not monotonic, the amplitude envelope decays slowly (like $1/\Omega$), and, consequently, the cutoff is gradual.

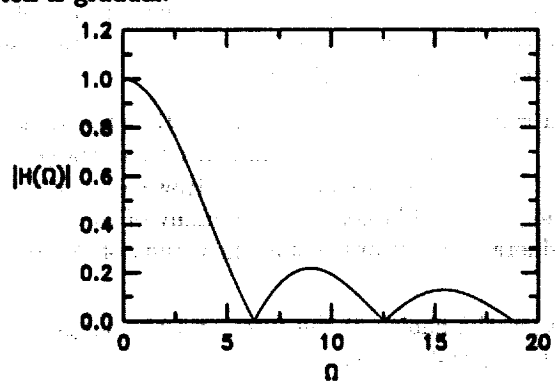

Figure 1. Transfer function of integral filter of Eq. 1.

Again by analogy to difference operators, to each filter operator is associated an order-property that quantifies the behavior of the filter as Ω tends toward zero. The order property is revealed by the leading non-zero term in Δ in the Taylor-series expansion of the filter. For example, for a function of time only, the Taylor-series expansion with respect to Δ of the filter of Eq. 1 is

$$F\left[f(t),\Delta\right] = f(t) - \frac{\Delta}{2}f'(t) + \frac{\Delta^2}{6}f''(t) + O(\Delta^3) \tag{2}$$

A class of causal time-domain filters more suitable for LES than Eq. 1 is that of the so-called Butterworth filters. Figure 2 compares the moduli of the transfer functions of prototypical Butterworth analog (BA) low-pass filters of orders 1, 2, and 4, each of which has a nominal cutoff frequency $\Omega_c' = 1$. The properties and design constraints of the first- and second-order low-pass BA prototypes can be found in Strum and Kirk[12]. The fourth-order BA prototype was developed by the author using Mathematica. The prototype BA filters shown in Fig. 2 are readily discretized and adapted to an arbitrary cutoff Ω_c.

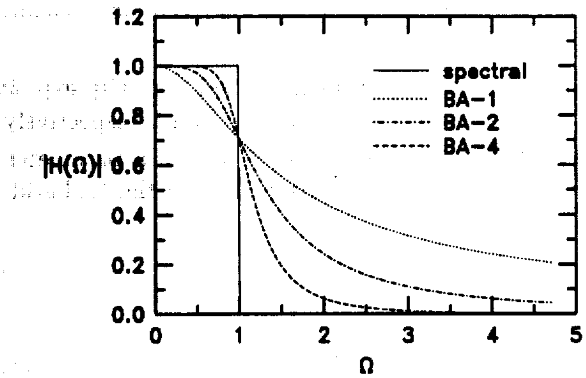

Figure 2. Comparison of Butterworth analog (BA) filters of orders 1, 2, and 4 with spectral (sharp cut-off) filter.

The order of a filter is closely related to the flatness of its transfer function at $\Omega = 0$. In general, for a filter of order n, $H^{(k)}(\Omega)|_{\Omega=0} = 0$ for all $0 < k \le n$, but $H^{(n+1)}(0) \neq 0$. In particular, Butterworth filters manifest several desirable properties for applications to LES: 1) stability, i.e., $|H(\Omega)| \le 1$; 2) their transfer functions decay monotonically with increasing Ω, and 3) their transfer functions are maximally flat near $\Omega = 0$. For comparison, Fig. 2 also presents the idealized transfer function of an analog spectral filter, which can be considered of infinite order.

For brevity in future discussions, let an overbar denote filtered quantities; that is, $\bar{f} \equiv F[f(t), \Delta]$ for some fixed Δ. From Figs. 1 and 2 it can be inferred that, except for the spectral filter, $\bar{\bar{f}} \neq \bar{f}$. It is also clear that the spectral ideal is more closely approximated as the order n increases. As a result, high-order temporal filters are problematic for practical applications to LES, because they necessitate the storage of relatively more time history. This may be a principal reason that, to date, time-domain filtering has been avoided by practitioners of LES, as hinted by Moin and Jimenez[8] in their survey paper.

One might naively assume (as did the author originally) that higher order is better. One of the more significant results of this work is to show that, in the context of LES, lower order filters are desirable for several reasons. This is particularly good news if one wants to consider the application of time-domain filters to LES, for example, as in Pruett[10].

To develop the results of the next sections, we make use of the Taylor series expansions of filter operators. For example, recall Eq. 1 above. More generally, assuming sufficient differentiability of u, any time-domain filter can be expanded as

$$\bar{u}(t, \vec{x}, \Delta) = u + c_1 \Delta u' + c_2 \Delta^2 u'' + c_3 \Delta^3 u''' + \dots \tag{3}$$

where primes denote temporal partial derivatives. For spatially multi-dimensional filter operators, similar expansions could be derived; however, their Taylor expansions would, of course, be multi-dimensional. In general, a filter is of order n in Δ provided $c_n \neq 0$, but $c_k = 0$ for $1 \le k < n$. Thus, Eq. 1, for example, defines a first-order filter. Similarly, one can show that both Gaussian and tophat physical-space filters are first order.

3 Conventional Dynamic SGS Modeling

In tensor notation, the linearly filtered Navier-Stokes equations are given by

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{4}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \nabla^2 \bar{u}_i + \frac{\partial \tau_{ij}}{\partial x_j} \tag{5}$$

where repeated indices imply summation, and τ_{ij} is the SGS (residual) stress tensor defined as

$$\tau_{ij} \equiv \bar{u}_i \bar{u}_j - \overline{u_i u_j} \tag{6}$$

Equation 6 is exact; inexactness enters only when the τ_{ij} are modeled. We focus now on dynamic modeling of the residual stresses.

Dynamic models[6] of the SGS stress tensor are rooted in the Germano[5] identity and typically exploit successive (spatial) "grid" and "test" filtering operations with associated length scales l and $\hat{l}$, respectively. To transition to temporal filtering, we explicitly assume that $\frac{\hat{\Delta}}{\Delta} = \frac{\hat{l}}{l} = r$, where $r > 0$ is a parameter. Typically, $r = 2$. The Germano identity relates the resolved turbulent stress tensor $\mathcal{L}_{ij}$ and the "subgrid" and "subtest" stress tensors, τ_{ij} and T_{ij}, respectively. Specifically,

$$\mathcal{L}_{ij} = T_{ij} - \hat{\tau}_{ij} \tag{7}$$

where

$$\mathcal{L}_{ij} \equiv \hat{\bar{u}}_i \hat{\bar{u}}_j - \widehat{\bar{u}_i \bar{u}_j} \tag{8}$$

and

$$T_{ij} \equiv \hat{\bar{u}}_i \hat{\bar{u}}_j - \widehat{\overline{u_i u_j}} \tag{9}$$

The Germano identity is exact; moreover, its left-hand side is computible. It remains to model each of the terms on the right-hand side, which is frequently accomplished via the Smagorinsky eddy-viscosity model, namely

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} \approx 2Cl^2|\bar{S}|\bar{S}_{ij} \tag{10}$$

Here, δ_{ij} is the Kronecker delta, $\bar{S}_{ij}$ is the resolved-scale strain-rate tensor, $|\bar{S}| \equiv \sqrt{2\bar{S}_{ij}\bar{S}_{ij}}$, l is the characteristic length scale associated with the grid filter, and C is the model constant of primary interest. In general, a criticism of eddy-viscosity models is their implicit assumption that the principal axes of the residual stress and the resolved-scale strain-rate tensors are aligned (Moin and Jiménez[8]). Indeed, in a recent experiment that examined turbulent three-dimensional boundary-layer flow, Compton and Eaton[4] found considerable misalignment between residual-stress and strain-rate tensors in the near-wall region. They concluded that eddy-viscosity models are inappropriate for such flows. Moin and Jimenez[8] propose a more generally applicable model, for which each stress-tensor component has its own coefficient, namely

$$\tau_{ij} \approx (C_{ik}\bar{S}_{kj} + C_{jk}\bar{S}_{ki})l^2|\bar{S}| = \frac{1}{2}(C_{ik}\beta_{kj} + C_{jk}\beta_{ki}) \tag{11}$$

where

$$\beta_{ij} \equiv 2l^2|\bar{S}|\bar{S}_{ij} \tag{12}$$

We will refer to Eq. 11 as the "generalized residual-stress model." The generalized residual-stress tensor is symmetric; hence, six independent coefficients must be determined. In principle, these coefficients can be uniquely determined by the dynamic procedure of Germano et al.[6], as adapted to the generalized model by Moin and Jimenez[8]. We briefly describe the procedure below.

It is now assumed that the subtest stresses can also be modeled by Eq. 11 with the same coefficients C_{ij}, in which case

$$T_{ij} \approx (C_{ik}\hat{\bar{S}}_{kj} + C_{jk}\hat{\bar{S}}_{ki})\hat{l}^2|\hat{\bar{S}}| \tag{13}$$

(In the author's opinion, this assumption represents a considerable leap of faith, given that τ_{ij} and T_{ij} are not formal twins.) From Eqs. 7, 11, and 13, a set of integral equations for the coefficients C_{ij} follows, namely

$$2\mathcal{L}_{ij} \approx (C_{ik}\alpha_{kj} + C_{jk}\alpha_{ki} - \widehat{C_{ik}\beta_{kj}} - \widehat{C_{jk}\beta_{ki}}) \tag{14}$$

where

$$\alpha_{ij} \equiv 2\hat{l}^2|\hat{\bar{S}}|\hat{\bar{S}}_{ij} \tag{15}$$

We now exploit the Taylor-series expansion of the test filter to obtain a simple approximation to Eq. 14. For specificity, we use the filter of Eq. 1 and its expansion Eq. 2. Omitting details for brevity, we obtain the approximation

$$2\mathcal{L}_{ij} \approx (r^2-1)\left(C_{ik}\beta_{kj} + C_{jk}\beta_{ki}\right) + 2L.O.E. + O(l^2\hat{\Delta}^2) = 2(r^2-1)\tau_{ij} + 2L.O.E. + O(l^2\hat{\Delta}^2) \qquad (16)$$

where the leading-order error (L.O.E.) term in $\hat{\Delta}$ is given by

$$L.O.E. = -\frac{\hat{\Delta}}{4}\left[r^2\left(C_{ik}\frac{\partial\beta_{kj}}{\partial t} + C_{jk}\frac{\partial\beta_{ki}}{\partial t}\right) - \left(\frac{\partial C_{ik}\beta_{kj}}{\partial t} + \frac{\partial C_{jk}\beta_{ki}}{\partial t}\right)\right] \qquad (17)$$

We conclude that

$$\tau_{ij} \approx \frac{\mathcal{L}_{ij}}{r^2-1} - \frac{L.O.E.}{r^2-1} + O(l^2\hat{\Delta}^2) \qquad (18)$$

We now assume that the highest order term is insignificant. It remains to show that the second term (L.O.E.) on the right-hand side of Eq. 18 is of lesser significance (on average) than the first. We assume here that all quantities have been previously scaled by appropriate reference values, so that we are dealing only with dimensionless quantities. It particular, lengths have been scaled by the wavelength of the largest eddies, and time has been scaled by the large-eddy turnover time. From Eqs. 8 and 2, to leading order in $\hat{\Delta}$, the resolved turbulent stresses are given by

$$\mathcal{L}_{ij} = \frac{\hat{\Delta}^2}{12}\left(\frac{\partial\overline{u}_i}{\partial t}\right)\left(\frac{\partial\overline{u}_j}{\partial t}\right) + O(\hat{\Delta}^3) \qquad (19)$$

In scaled variables, on the basis of reasonable assumptions and approximations (omitted for brevity), it can be argued that the L.O.E. above is relatively unimportant whenever the dimensionless frequency $\Omega \equiv \hat{\Delta}\omega << 1$ (provided the filter is of first- or second-order, as will be shown). Specifically, for example, if $\Omega_c \approx 0.1$, an entirely reasonable value in practice, then, Eq. 18 is approximated simply as

$$\tau_{ij} \approx \frac{\mathcal{L}_{ij}}{r^2-1} \qquad (20)$$

Remarkably, Liu et al.[7] arrive at a result similar to Eq. 20 from *experimental* measurements in a turbulent jet. Specifically, they obtain several components of the SGS stress tensor of a jet by two-dimensional particle velocimetry. Whereas eddy-viscosity closures correlate poorly with the measured residual stresses, the resolved stresses correlate well. They propose the simple stress-similarity model

$$\tau_{ij} = c_L\mathcal{L}_{ij} \qquad (21)$$

where the coefficient c_L is empirically derived. For $r = 2$, they obtain $c_L = 0.45 \pm 0.15$ for a "clipped" (no backscatter) SGS model by matching the exact and modeled SGS dissipation rates. For a model without clipping the optimal coefficient is approximately unity (Meneveau, personal communication). Either way, their result corroborates our observation that the residual and resolved turbulent stresses should be highly correlated.

The implications of our present results for the practice of dynamic SGS modeling are both troubling and hopeful. Because the effects of numerator and denominator of the modeled residual stresses essentially "cancel" in the present analysis, dynamic SGS models, viewed in the present light, are ultimately *independent of the form of their underlying model* (whether Smagorinsky or otherwise)! This unanticipated result suggests that the whole concept of dynamic modeling needs re-examination.

In hindsight, it appears to the author that the basis of dynamic models in the Germano identity is fundamentally flawed. The Germano "identity" is actually tautological, having been derived simply by regrouping and renaming certain quantities from the starting point $\hat{\tau}_{ij} = \hat{\tau}_{ij}$. This is not to say that the idea of dynamic modeling is flawed, only that there is no necessity for the Germano identity, as will be shown. Moreover, not only is the Germano identity unnecessary, it results in the practical difficulty associated with the vanishing denominator of the model coefficient.

4 Alternate Approach to Dynamic Modeling

In light of the discussion above, it is natural to ask: *Can the residual stresses be modeled by the resolved turbulent stresses without appealing to the Germano identity?*

By applying the general Taylor expansion Eq. 3 to Eq. 6, we obtain

$$\tau_{ij} = (c_1^2 - 2c_2)u_i'u_j'\Delta^2 + (c_1c_2 - 3c_3)(u_i'u_j'' + u_j'u_i'')\Delta^3 + \text{H.O.T.} \tag{22}$$

where H.O.T. denotes higher order terms. Because the SGS-stress tensor τ_{ij} arises solely from the quadratic nonlinearity of the NS equations, it is quadratic at leading order in Δ, *provided that the filter is of either first- or second-order.* On the other hand, if the filter is of order $n > 2$, then τ_{ij} is of leading order n. Because the Smagorinsky model is of second-order in l (or equivalently, in Δ), it can be concluded immediately from Eqs. 10 and 22 that the model is appropriate only in the context of first- or second-order filters. Moreover, the use of Smagorinsky-based SGS models is totally inconsistent with spectral filters (which as we have said previously, can be considered of infinite order). This conclusion should hold regardless of whether filtering is accomplished in space or in the time domain. Our results are supported by experimental evidence. Liu et al.[7] find high correlations between τ_{ij} and $\mathcal{L}_{ij}$ when filtering is accomplished consistently with either Gaussian or physical-domain top-hat filters (both of which are first-order in our terminology). On the other hand, negligible correlations exist when a sharp cut-off filter is used in Fourier space (i.e., a spectral filter in our terminology).

Of fundamental importance in dynamic models is the resolved turbulent stress tensor $\mathcal{L}_{ij}$ (Eq. 8). As mentioned previously, the resolved turbulent stresses can be directly computed by filtering the resolved velocity fields $\bar{u}_i$. Let us now expand $\mathcal{L}_{ij}$ analogously to Eq. 22 above. To this end, we presume that the grid and test filters differ only in their respective filter widths. More precisely, if the grid filter is defined by Eq. 3, then the test filter is defined by

$$\hat{u}(t,\vec{x}) \equiv F\left[u(t,\vec{x}),\hat{\Delta}\right] = u + c_1(r\Delta)u' + c_2(r\Delta)^2u'' + c_3(r\Delta)^3u''' + \ldots \tag{23}$$

with the same coefficients c_i as in Eq. 3. From Eqs. 3, 8, and 23, and with the aid of Mathematica, it follows that

$$\mathcal{L}_{ij} = (c_1^2 - 2c_2)u_i'u_j'(r\Delta)^2 + (c_1^3r^2 - 2c_1c_2r^2 + c_1c_2r^3 - 3c_3r^3)(u_i'u_j'' + u_j'u_i'')\Delta^3 + \text{H.O.T.} \tag{24}$$

From comparisons of Eq. 22 and Eq. 24, we conclude that the SGS stresses can be approximated to leading order by the resolved stresses scaled by r^2; that is,

$$\tau_{ij} \approx \frac{\mathcal{L}_{ij}}{r^2} \tag{25}$$

How good is the approximation? Let the approximation error E be defined

$$E_{ij} \equiv \tau_{ij} - \frac{\mathcal{L}_{ij}}{r^2} \tag{26}$$

From Eqs. 22 and 24, we obtain

$$E_{ij} = \left[3c_3(r-1) + c_1c_2(3-r) - c_1^3\right]\Delta^3(u_i'u_j'' + u_j'u_i'') + \text{H.O.T.} \tag{27}$$

From Eqs. 22 and 27, we immediately conclude the following:

1. If the filter is of either first- or second-order in Δ, then the approximation error is of higher order (3) than is the SGS stress (2), and the approximation is likely to be reasonably accurate (given additional constraints to be addressed shortly).

2. On the other hand, for any filter of order $n > 2$, the approximation error is of the same order as τ itself; hence, τ is likely to vanish in the noise of the approximation.

3. If the filter is of order two ($c_1 = 0.0$, $c_2 \neq 0.0$) then

$$E_{ij} = 3c_3(r-1)\Delta^3(u_i'u_j'' + u_j'u_i'') + \text{H.O.T.} \tag{28}$$

4. Far from being inadmissible, as implied by the conventional dynamic modeling approach, $r = 1$ is *optimal* for second-order filters in that the leading error term vanishes.

5. Because the residual stresses can be approximated directly from the resolved turbulent stresses, the Germano identity is unnecessary for the development of dynamic SGS models.

5 Discussion

Although, for brevity, the present results were derived using time-domain filtering, similar results could have been obtained for spatial filtering, albeit by more arduous mathematics. For example, Eq. 19 is the time-filtered analog to the space-filtered result of Clark et al.[3] as interpreted by Speziale[11], who reports that

$$\mathcal{L}_{ij} = \frac{\hat{\Delta}^2}{12}\left(\frac{\partial \overline{u}_i}{\partial x_k}\right)\left(\frac{\partial \overline{u}_j}{\partial x_k}\right) + O(\hat{\Delta}^3) \tag{29}$$

Although Eqs. 20 and 25 are similar and in reasonable agreement for moderately large r, they are not identical. Whence the difference? Because the former originates from the Germano identity and the latter explicitly avoids it, we speculate that the discrepancy follows from the assumption that τ_{ij} and T_{ij} of Eq. 7 can both be modeled by formally identical models, despite some formal dissimilarity.

If both analysis and experiment conclude that the residual stresses correlate closely with the (computible) resolved stresses, then it is tempting to suggest for LES the use of SGS models that contain only scale-similarity terms. However, it is well known (e.g., Liu et al.[7]), that scale-similarity models alone are insufficiently dissipative, and such calculations are almost guaranteed to blow up, particularly if the numerical scheme is non-dissipative. Our interpretation of the situation is as follows: the SGS models of LES must unfortunately play two roles: one physical and one mathematical. Whereas scale-similarity models appear sufficient to capture the physics of SGS energy transfer, additional dissipation (e.g., a Smagorinsky-like term) is necessary for mathematical reasons; i.e., to stabilize the numerical scheme whenever resolution is marginal. These roles are somewhat separated by mixed models (e.g., Bardina[2]), which include both scale-similarity and dissipative terms.

Although our results are completely consistent with the experimental results of Liu et al.[7], they are only partially consistent with the DNS results of Piomelli et al.[9], whose *a priori* tests show good agreement between modeled and exact stresses both for a mixed model with a Gaussian filter and for the Smagorinsky model with a sharp cut-off filter. Whereas the former result is consistent with our findings, the latter is not. However, as Piomelli et al.[9] are careful to point out: "The fact that the SGS stress is essentially zero when the cutoff filter is used on the present [DNS] grid indicates that, with that filter, the grid may be capable of resolving the Reynolds stress and no model is needed." Thus, the inconsistency may be more apparent than actual. We are currently conducting *a priori* tests to further validate our present analysis.

6 Conclusions

1. Mathematically tautological, the Germano "identity" is suspect as a basis for dynamic SGS modeling.
2. A practical difficulty with dynamic SGS modeling, manifested in the vanishing denominator of the model coefficient, is directly attributable to the use of the Germano identity.
3. The Germano identity is not only problematic, it is an unnecessary basis for dynamic SGS models.
4. For first- or second-order filter operators, the computible resolved turbulent stresses, when properly scaled, closely approximate the residual stresses, without appeal to the Germano identity.

5. In general, filters of higher than second order are inconsistent with the Smagorinsky SGS model.
6. In particular, spectral filters are inconsistent with the Smagorinsky SGS model.
7. In LES, the SGS model plays two roles: one physical and one mathematical. To separate these roles, mixed models should be exploited. In mixed models, the scale-similarity term captures the physics and the dissipative term prevents numerical instability. Common experience with LES reveals that the scale-similarity term alone is insufficient.
8. The scaling of the scale-similarity term of mixed models should depend on the choice of the parameter r relating grid and test filter widths. This has been overlooked in practice.
9. A new model for the dissipative term, directly based on the computible resolved turbulent stresses, is sorely needed.

Acknowledgements

The author is grateful to Drs. Gordon Erlebacher of Florida State University; Kristine Meadows, Craig Streett, Michele Macaraeg, and Bart Singer of NASA Langley Research Center; and Bassam Younis of University College, London, for helpful discussions. He is also grateful to Dr. Sandip Ghosal of Los Alamos National Laboratory, to Dr. Klaus Adams of ZTH in Switzerland, and to Prof. Charles Meneveau of Johns Hopkins University for beneficial e-mail discussions and references regarding dynamic SGS modeling. Special thanks is due Prof. Ugo Piomelli of the University of Maryland for his time, insights, suggestions, clarifications, and numerous references. Any confusion that remains is entirely the author's.

References

[1] A. A. Aldama, *Filtering Techniques for Turbulent Flow Simulation*, Springer-Verlag, Berlin, 1990.

[2] J. Bardina, J. H. Ferziger, and W. C. Reynolds, "Improved Subgrid Scale Models for Large Eddy Simulation," AIAA Paper No. 80-1357, 1980.

[3] R. A. Clark, J. H. Ferziger, and W. C. Reynolds, "Evaluation of Subgrid-Scale Models Using an Accurately Simulated Turbulent Flow," *J. Fluid Mech.*, Vol. 91, 1979, pp. 1-16.

[4] D. A. Compton and J. A. Eaton, "Development of Near-Wall Statistics in a Three-Dimensional Turbulent Boundary Layers," in *Three-Dimensional Boundary Layers*, FED-Vol. 237, ASME, 1996.

[5] M. Germano, "Turbulence: The Filtering Approach," *J. Fluid Mech.*, Vol. 238, 1992, pp. 325-336.

[6] M. Germano, U. Piomelli, P. Moin, and W. H. Cabot, "A Dynamic Subgrid-Scale Eddy Viscosity Model," *Phys. Fluids A*, Vol. 3, 1991, pp. 1760-1765.

[7] S. Liu, C. Meneveau, and J. Katz, "On the Proporties of Similarity Subgrid-Scale Models as Deduced from Measurements in a Turbulent Jet," *J. Fluid Mech.*, Vol. 275, No. 83, 1994, pp. 83-119.

[8] P. Moin and J. Jimenez, "Large-Eddy Simulation of Complex Turbulent Flows," AIAA Paper No. 93-3099, 1993.

[9] U. Piomelli, P. Moin, and J. H. Ferziger, "Model Consistency in Large-Eddy Simulation of Turbulent Channel Flow," *Phys. Fluids A*, Vol. 31, No. 7, 1988, pp. 1884-1891.

[10] C. D. Pruett, "Time-Domain Filtering for Spatial Large-Eddy Simulation," to be presented at the 3rd Symposium on Transitional and Turbulent Compressible Flows, Annual Summer Meeting of the Fluids Engineering Division of the ASME, Vancouver, BC, June 22-26, 1997.

[11] C. G. Speziale, "Galilean Invariance of Subgrid-Scale Stress Models in the Large-Eddy Simulation of Turbulence," *J. Fluid Mech.*, Vol. 156, 1985, pp. 55-62.

[12] R. D. Strum and D. E. Kirk, *First Principles of Discrete Systems and Digital Signal Processing*, Addison-Wesley, New York, 1988.